Final Exam

June 2, 2023

Given time: 2 hours and 45 minutes (10h15 to 13h00)

Problem 1
Speckle
(2 points out of 6)

Consider the 4F imaging system shown in Figure 1. A diffuser is placed at the object plane and it is illuminated with a spatially uniform monochromatic plane wave. We describe the transmittance of the diffuser as a thin phase object with transmittance $t(x) = e^{-j\varphi(x)}$ where $\varphi(x)$ is a random function. $t(x)$ is given in file "diffuser.mat". You can neglect in this problem the finite apertures of the two lenses. Instead, an aperture with diameter A is placed at the middle plane of the 4F system.

a. Use BPM to calculate and plot the intensity of the light at the output plane for A=250 μm, λ=532 nm, F=37.5 mm
b. Repeat the experiment for A=500 μm.
c. Measure to the best of your ability the typical speckle size in the intensity pattern you observe. How does this compare to the spatial resolution of the 4F imaging system?
d. Speckle is a phenomenon that arises only with coherent light. You will verify this by repeating the calculation of the output intensity when the random diffuser is illuminated with a spatially incoherent, quasi monochromatic light $i(x,t) = a(x,t)\ e^{j(\omega t - kz)}$ where $a(x,t)$ is a random function of x and t describing the amplitude of the light incident on the diffuser. You can implement this in python by averaging N_{avg} BPM simulations where the input is modulated by a random noise using the code in Lecture 5. For the noise use the following function:

```
noise=np.exp(1j*np.random.normal(0, p_coh, u.shape))
```

where `p_coh` is the standard deviation of the noise. You can use this parameter to model in an indirect way the beam coherence length. Use BPM to calculate and plot the light intensity at the image plane for `p_coh=1.0` and `p_coh=2.0`. Compare with part a and comment on the results.

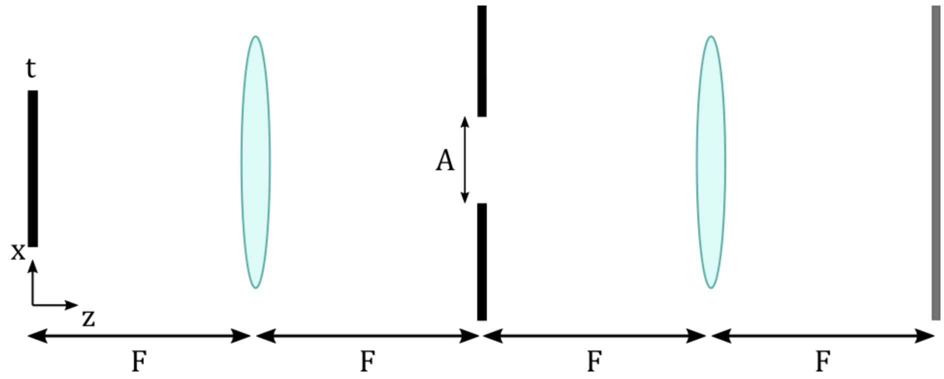

Figure 1

Simulation parameters:

```
Nx = Ny = Nz = 512
Lx = Ly = 2 mm
Lz = 150 mm
n0 = 1.0
λ = 532nm
Navg = 200
```

Problem 2
Digital confocal microscopy
(2 points out of 6)

Confocal microscopy allows us to form 3D images of the transmittance by scanning in 3D the object while illuminating the object with a focused beam (see Figure 2a). A pinhole is placed at the image plane of the illuminating focused beam. The scattered light emanating from the spot of the focused illuminating beam is imaged onto the pinhole and measured through a photodetector. Scattering from other parts of the 3D sample miss the pinhole (at least partially) and they are not detected. In this problem you will explore the 3D sectioning of ***digital*** confocal microscopy. You will work in 2D (x-z) to save computational time.

a. We provide you with a series of holographically measured fields by a camera (in correspondence of the blue plane in Figure 2b) for different object positions (the object is moved along the blue directions in Figure 2). Describe in words how to propagate in the digital computer the field at P_1 through an imaging lens (see Figure 2b). Download the notebook "Final_Ex2.ipynb" and the file "confocal.mat" which contains 4800 acquired fields (in the variable 'meas') and the z,x position of the object for each of them. Use BPM to propagate the field stored in position 1800 for a distance $z = 2F$ until P_2 and show the field distribution in zx, what do you notice?
b. Now use two for loops (already initialized in the notebook) to scan over all the x-z positions. For each position you will propagate the provided measured fields until P_2 and integrate the ***intensity*** over all the camera pixels.
c. Place a virtual pinhole with aperture diameter 2 µm at plane P_2. Calculate and plot the light transmitted through the pinhole as a function of the position of the sample in x and z. Compare the x-z image you obtain with the one in b. Comment on the effects of the pinhole in your imaging system.

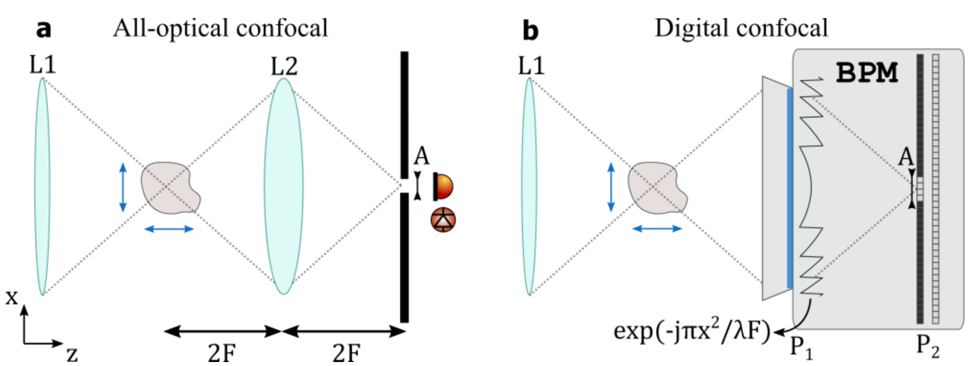

Figure 2

Simulation parameters:

```
Nx = Nz = 256
```

```
Lx = 200 µm
Lz = 400 µm
n0 = 1.0
λ = 680 nm
```

Problem 3
Benton holography
(2 points out of 6)

A hologram of a planar object O is recorded on photographic film at a distance z_1 with a plane wave reference at an angle θ_x (R_1) with respect to the z-axis. Assume that the transmittance of the recorded hologram is proportional to the incident light intensity during the recording.

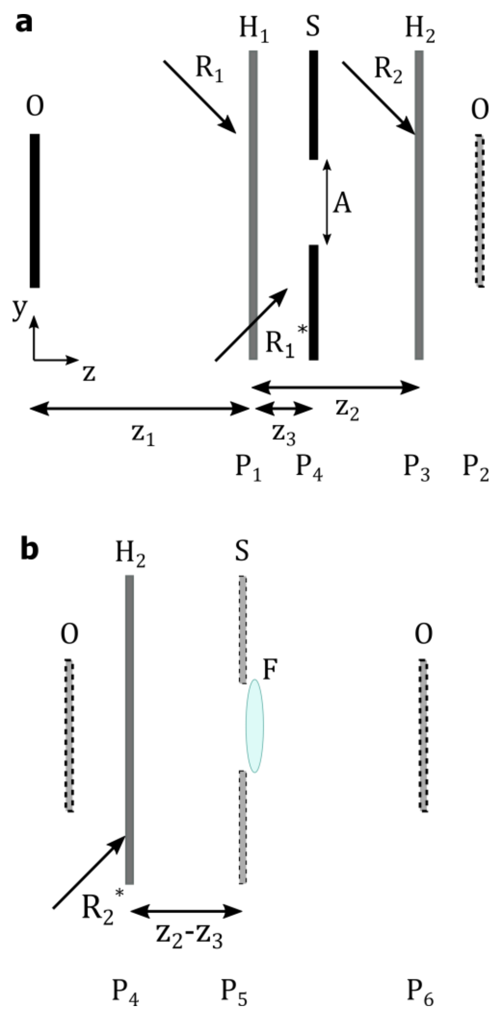

Figure 3

a. Show with pencil and paper that when the hologram is illuminated with a coherent plane wave at an angle $-\theta_x$ (R_1^*) as shown in the figure a REAL image of the input image O forms at a distance z_1 after the hologram (P_2). We provide you this hologram recorded in P_1 with a blazed grating $R_1 = \exp[jk\sin\theta_x\, x]$ with $\theta_x = 20°$

in the file "hologram.mat". Find the object O in P_2 with BPM by illuminating the hologram with R_1^*.

b. A second hologram is recorded by interfering the reconstruction of the first hologram as described in part a with a with a second reference beam $R_2 = \exp[jk \sin\theta_y\, y]$ with $\theta_y = 2°$. The photographic film is placed at a distance $z_2 < z_1$ after the first hologram (see Figure 3a), so in a plane P_3 before the one where the object is reproduced (P_2). Furthermore, a rectangular slit infinitely extended along x and with a width A along y is placed during the recording at a distance $z_3 < z_2$ (P_4) after the first hologram (see figure 3a). The second hologram is reconstructed with a plane wave at an angle $-\theta_y$ (R_2^*), see Figure 3b. Show with BPM that a slit is formed in P_5 at a distance $z_2 - z_3$.

c. Place now a lens in P_5 and find an adequate focal lens and the position of P_6 where you should see the object O. Return the result of the field you computed in P_6 with BPM.

[**BONUS**]: The idea of rainbow holography is that if you place a lens (your "eye") at the position of the slit you will see the object. If you move up and down you will lose the object because the lens of the eye misses the slit. However, if we use a white light source we can observe the object at the expense of losing the vertical parallax. Explain this effect.

[**Hint**]: The parameters have been chosen in a way that you can perform free space propagation in a single step. So, if you want to save time, avoid for loops for free space propagation.

Simulation parameters:

```
Nx = 512
Lx = 10 mm
Lz = 600 mm
n0 = 1.0
λ = 532 nm
A = 600 µm
z1 = 600 mm
z2 = 480 mm
z3 = 120 mm
```